GRADE 12 DIPLOMA EXAMINATION

Mathematics 30

January 1987

Alberta
EDUCATION

GRADE 12 DIPLOMA EXAMINATION
MATHEMATICS 30

DESCRIPTION

Time: 2½ hours

Total possible marks: 65

This is a **CLOSED-BOOK** examination consisting of two parts:

PART A: 52 multiple-choice questions each with a value of 1 mark.

PART B: Three written-response questions for a total of 13 marks.

A mathematics data booklet is provided for your reference. Approved calculators may be used.

GENERAL INSTRUCTIONS

Fill in the information on the answer sheet as directed by the examiner.

For multiple-choice questions, read each carefully and decide which of the choices BEST completes the statement or answers the question. Locate that question number on the answer sheet and fill in the space that corresponds to your choice. USE AN HB PENCIL ONLY.

Example

This examination is for the subject area of

A. Chemistry
B. Biology
C. Physics
D. Mathematics

Answer Sheet

A	B	C	D
①	②	③	●

If you wish to change an answer, please erase your first mark completely.

For written-response questions, read each carefully, show all your calculations, and write your answer in the space provided in the examination booklet.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

DO NOT FOLD EITHER THE ANSWER SHEET OR THE EXAMINATION BOOKLET.

The presiding examiner will collect the answer sheet and examination booklet for transmission to Alberta Education.

JANUARY 1987

PART A

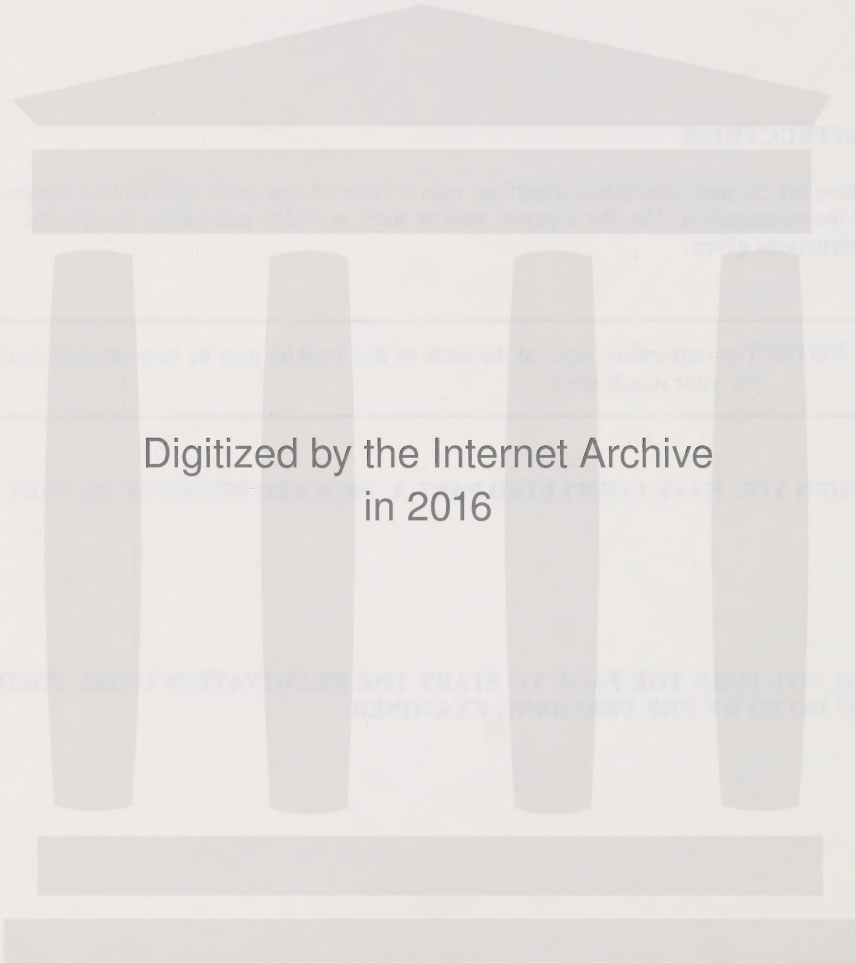
INSTRUCTIONS

There are 52 multiple-choice questions with a value of one mark each in this section of the examination. Use the separate answer sheet provided and follow the specific instructions given.

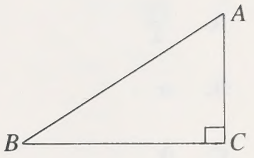
NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

WHEN YOU HAVE COMPLETED PART A, PROCEED DIRECTLY TO PART B.

DO NOT TURN THE PAGE TO START THE EXAMINATION UNTIL TOLD TO DO SO BY THE PRESIDING EXAMINER.



1. In the diagram at the right, $\cos A$ is



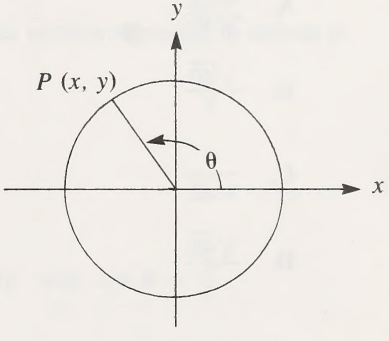
A. $\frac{AC}{AB}$

B. $\frac{AC}{BC}$

C. $\frac{BC}{AB}$

D. $\frac{BC}{AC}$

2. If $P(x, y)$ is a point on the unit circle shown to the right, then $\csc \theta$ is



A. $\frac{x}{y}$, $y \neq 0$

B. $\frac{y}{x}$, $x \neq 0$

C. $\frac{1}{x}$, $x \neq 0$

D. $\frac{1}{y}$, $y \neq 0$

3. The range of the function $y = 3 \sin (2\theta - \pi)$ $\theta \in R$ is

A. R

B. $y \leq 3, y \in R$

C. $-1 \leq y \leq 1, y \in R$

D. $-3 \leq y \leq 3, y \in R$

4. The solution to the equation $2 + 3 \cos \theta = -1, 0 \leq \theta < 2\pi$ is

A. $\frac{3\pi}{2}$

B. π

C. 0

D. -1

5. If $\sin A = \frac{3}{7}, 0 < A < \frac{\pi}{2}$, then $\cot A$ is equal to

A. $\frac{3}{\sqrt{40}}$

B. $\frac{\sqrt{40}}{3}$

C. $\frac{7}{\sqrt{40}}$

D. $\frac{\sqrt{40}}{7}$

6. If $\tan^2 \theta$ is expressed in terms of $\sin^2 \theta$, it is equivalent to

A. $\frac{\sqrt{1 - \sin^2 \theta}}{\sin^2 \theta}$

B. $\frac{1 - \sin^2 \theta}{\sin^2 \theta}$

C. $\frac{\sin^2 \theta}{1 - \sin^2 \theta}$

D. $\frac{1}{1 - \sin^2 \theta}$

7. The value of $\tan\left(\frac{\pi}{5}\right)$ is equal to

A. $-\tan\left(-\frac{\pi}{5}\right)$

B. $-\tan\left(\frac{\pi}{5}\right)$

C. $\tan\left(-\frac{\pi}{5}\right)$

D. $\frac{1}{\cot\left(-\frac{\pi}{5}\right)}$

8. A wheel turns 36 000 degrees in one direction. This rotation expressed in radians is

A. 100π
B. 180π
C. 200π
D. 360π

9. If C is a third quadrant angle and $C = (90 + 2B)°$ with $\cos B = \frac{1}{2}$, then $\sec C$ is equal to

A. $-\frac{1}{\sqrt{3}}$

B. $-\frac{2}{\sqrt{3}}$

C. $-\frac{1}{4}$

D. $-\frac{1}{2}$

10. In ΔRST, if $\angle R = 60°$, $\angle S = 40°$, and $r = 30$ cm, then the best approximation to the measure of t is

A. 40 cm
B. 39 cm
C. 36 cm
D. 34 cm

11. If each side of an equilateral triangle is 80 cm long, then the area of the triangle is

A. 1600 cm^2

B. $\frac{3200}{\sqrt{3}} \text{ cm}^2$

C. 3200 cm^2

D. $1600\sqrt{3} \text{ cm}^2$

12. The slope of a line determined by the equation $3x - 4y = 7$ is

A. $\frac{3}{4}$

B. $-\frac{3}{4}$

C. 3

D. -3

13. The equation of the circle $4x^2 - 8x + 4y^2 + 24y + 4 = 0$ expressed in the form $(x - h)^2 + (y - k)^2 = r^2$ is

A. $(x - 1)^2 + (y + 3)^2 = 3$

B. $(2x - 1)^2 + (2y + 3)^2 = 3$

C. $(x - 1)^2 + (y + 3)^2 = 9$

D. $(2x - 1)^2 + (2y + 3)^2 = 9$

14. The equation of a circle with centre (2, 6) and radius 4 is

A. $x^2 + y^2 + 4x + 12y + 36 = 0$

B. $x^2 + y^2 + 4x + 12y + 24 = 0$

C. $x^2 + y^2 - 4x - 12y + 24 = 0$

D. $x^2 + y^2 - 4x - 12y + 36 = 0$

15. Any point on the curve defined by $x^2 - 20y = 0$ is equidistant from

A. $(0, -5)$ and $(-5, 0)$
B. $(0, 5)$ and $(5, 0)$
C. $(5, 0)$ and the line $y = 5$
D. $(0, 5)$ and the line $y = -5$

16. If a parabola has its focus at $(-2, -3)$ and directrix $x = 4$, then the vertex is at

A. $(3, -3)$
B. $(1, -3)$
C. $(0, -3)$
D. $(-3, -3)$

17. The foci and the length of the minor axis of the ellipse defined by the equation $49y^2 + 9x^2 = 441$ are

A. $(\pm\sqrt{40}, 0)$ and 6

B. $(0, \pm\sqrt{40})$ and 14

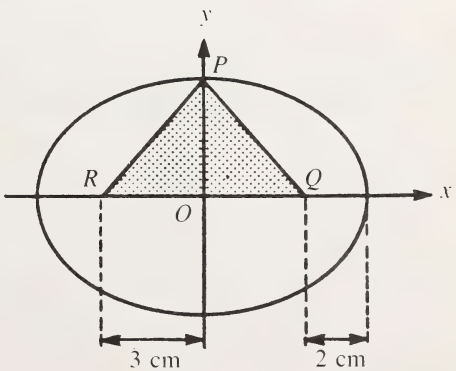
C. $(\pm\sqrt{58}, 0)$ and 6

D. $(0, \pm\sqrt{58})$ and 14

18. Which of the following statements based on the graph of $\frac{x^2}{4} + \frac{y^2}{8} = 1$ is TRUE?

A. The major axis is vertical.
B. The minor axis is vertical.
C. The foci are located on the horizontal axis.
D. The major axis is twice as long as the minor axis.

19. An isosceles triangle PQR is drawn inside an ellipse as shown at the right. If R and Q are the foci of the ellipse, the area of the triangle is



A. 24 cm^2

B. 15 cm^2

C. 12 cm^2

D. 6 cm^2

20. If a hyperbola passes through $(5, 2.25)$ and has foci at $(\pm 5, 0)$, its equation is

A. $\frac{y^2}{9} - \frac{x^2}{16} = 1$

B. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

C. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

D. $\frac{y^2}{16} - \frac{x^2}{9} = 1$

21. A hyperbola with one focus at $(0, -6)$ and conjugate axis 4 units long has an equation of

A. $\frac{y^2}{20} - \frac{x^2}{16} = 1$

B. $\frac{y^2}{52} - \frac{x^2}{16} = 1$

C. $\frac{y^2}{40} - \frac{x^2}{4} = 1$

D. $\frac{y^2}{32} - \frac{x^2}{4} = 1$

22. The sequence $4, 2, 1, \frac{1}{2}, \ldots, 8(2^{-n}), \ldots$ is

A. infinite and arithmetic
B. infinite and geometric
C. finite and arithmetic
D. finite and geometric

23. If $x + 2$, $3x - 4$, and $5x - 10$ are consecutive terms of an arithmetic sequence, the value of x such that the sequence has a common difference of 2 is

A. 0
B. 2
C. 4
D. 8

24. The sum of the arithmetic series $-2 + 1 + 4 + 7 + \ldots + 61$ is

A. 530
B. 588
C. 649
D. 713

25. If the 13th and 14th terms of an arithmetic sequence are 71 and 77 respectively, the 6th term is

A. 23
B. 29
C. 35
D. 41

26. If $x - 2$, x, and $x + 4$ are consecutive terms of a geometric sequence, then the value of x is

A. 2
B. 4
C. 6
D. 8

27. A sum of money invested at 9% per annum compounded semi-annually amounts to \$12 000 in 8 years. This sum is

A. $\dfrac{12\,000}{(1.045)^8}$

B. $\dfrac{12\,000}{(1.09)^8}$

C. $\dfrac{12\,000}{(1.045)^{16}}$

D. $\dfrac{12\,000}{(1.09)^{16}}$

28. The series $(1)(3) + (3)(7) + (5)(15) + (7)(31)$ can be written as

A. $\sum_{k=1}^{4} (2k - 1)(4k - 1)$

B. $\sum_{k=1}^{4} (2k - 1)(2^{k+1} - 1)$

C. $\sum_{k=1}^{4} (2^{k+1} - 1)$

D. $\sum_{k=1}^{4} (4k - 1)$

29. The $\lim_{a \to \infty} \left(\frac{16a^4 + 65}{a^4} \right)$ is

A. non-existent
B. 81
C. 65
D. 16

30. The limit of the convergent sequence $\frac{1}{2}, \frac{4}{7}, \frac{3}{5}, \ldots, \frac{2n}{3n + 1}, \ldots$ is

A. $\frac{3}{4}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. 0

31. If the sum of an infinite geometric series is 24 and the common ratio is $\frac{5}{6}$, then the first term is

A. 20
B. 6
C. 4
D. -4

32. The sum of the geometric series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \ldots$ is

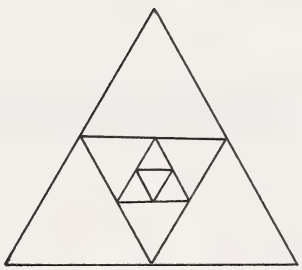
A. 3

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. 0

33. The midpoints of the sides of an equilateral triangle are joined as shown in the diagram to the right. This process is continued indefinitely. If each side of the largest triangle is 10 cm, the sum of all perimeters is



A. 120 cm
B. 100 cm
C. 75 cm
D. 60 cm

34. For the set of data 10, 9, 6, 9, 11, 12 the mean, median, and mode respectively are

A. 9, 9.5, 9
B. 9, 9, 9.5
C. 9.5, 9, 9.5
D. 9.5, 9.5, 9

35. The mean of the data in the table to the right is

A. 25.5
B. 26.0
C. 27.5
D. 28.0

Interval	Frequency
0.5 – 10.5	2
10.5 – 20.5	4
20.5 – 30.5	9
30.5 – 40.5	7
40.5 – 50.5	3

36. An automobile tire has a mean life of 64 000 km with a standard deviation of 3200 km. In a purchase of 1500 tires, the number lasting less than 54 400 km would be

A. 2
B. 38
C. 75
D. 88

37. A population has a normal distribution with mean μ and standard deviation σ. The per cent of the population that lies between $\mu - 2\sigma$ and $\mu + \sigma$ is

A. 95.0
B. 81.9
C. 68.0
D. 47.5

38. A balanced coin is flipped 400 times. Assuming a standard deviation of 10, the probability of getting from 215 to 225 heads is

A. 0.06
B. 0.43
C. 0.49
D. 0.93

39. A manufacturer advertises that his slide projector bulbs have a mean life of 24 h with a standard deviation of 2.5 h. Assuming a normal distribution, the probability that a bulb will last between 21 h and 30 h is

A. 0.88
B. 0.49
C. 0.38
D. 0.11

40. The expression $\dfrac{(a^2)^x \bullet (a^3)^y}{a^{x+y} \bullet a^{x-y}}$ is equivalent to

A. a^{3y}

B. a^{2x+y}

C. a^{2x+3y}

D. a^{4x+3y}

41. An exponential form of $\log_R\left(\dfrac{1}{M}\right) = -H$ is

A. $(-H)^M = \dfrac{1}{M}$

B. $R^{-H} = M$

C. $R^H = \dfrac{1}{M}$

D. $R^H = M$

42. If $5^{x+1} = 7$, then x is

A. $\dfrac{\log(7) - \log(5)}{\log(7)}$

B. $\dfrac{\log(7) - \log(5)}{\log(5)}$

C. $\dfrac{\log(5) - \log(7)}{\log(7)}$

D. $\dfrac{\log(5) - \log(7)}{\log(5)}$

43. If $\left(\frac{1}{2}\right)^{a+b} = 16$ and $\log_{(a-b)} 8 = -3$, then the values of a and b respectively are

A. $-\frac{7}{4}, -\frac{9}{4}$

B. $-\frac{5}{2}, -\frac{3}{2}$

C. $-4, 0$

D. $3, 1$

44. The approximate value of x in the equation $17^{x-1} = 12^{x+3}$ is

A. 10.6
B. 12.6
C. 29.5
D. 31.5

Use the following information to answer question 45.

Loudness of Sound

$L = 10 \log\left(\frac{I}{I_o}\right)$ where L = loudness of sound (decibels)
I = intensity of sound
I_o = minimum intensity audible to the human ear

45. If the intensity of a certain sound is 800 times the minimum intensity, its loudness measured in decibels is

A. 104
B. 36
C. 29
D. 17

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46. The line $x = -3$ intersects the graph of $y = x^3 - 11$ at the point

A. $(2, -3)$
B. $(-2, -3)$
C. $(-3, 16)$
D. $(-3, -38)$

47. If the polynomial $ax^m + bx^n + cx^p + d$ is written in descending order of degree, the statement that is always true is

A. $m + p > n + p$
B. $a + b > c + d$
C. $a > b$
D. $m > a$

48. If $x^3 + 9x^2 + 21x + 24$ is divided by $x + 2$, the coefficient of x in the quotient is

A. 7
B. 12
C. 14
D. 24

49. For what value of k will the remainder be 1 when $2x^3 + x^2 - 3kx + 7$ is divided by $x + 2$?

A. $\frac{5}{6}$

B. 1

C. $\frac{13}{3}$

D. $\frac{9}{2}$

50. One factor of $x^3 - x^2 - 10x - 8$ is

A. $x + 4$
B. $x - 1$
C. $x - 2$
D. $x - 4$

51. The x-intercepts of the graph of the polynomial function defined by $P(x) = 6x^3 - 13x^2 + x + 2$ are

A. $\frac{1}{2}, -\frac{1}{3}, 2$

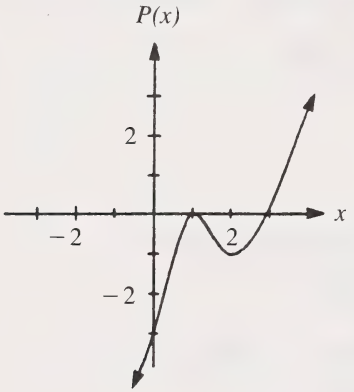
B. $2, -3, \frac{1}{2}$

C. $-\frac{1}{2}, \frac{1}{3}, -2$

D. $-2, 3, -\frac{1}{2}$

52. The sketch of the graph of the polynomial $P(x) = (x - 1)^2 (x - 3)$ is

A.



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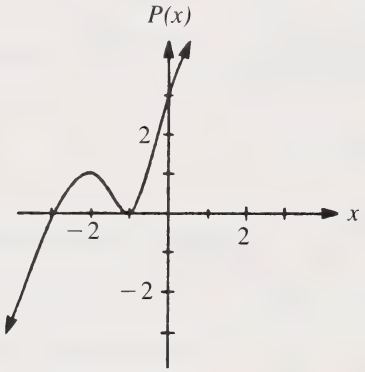

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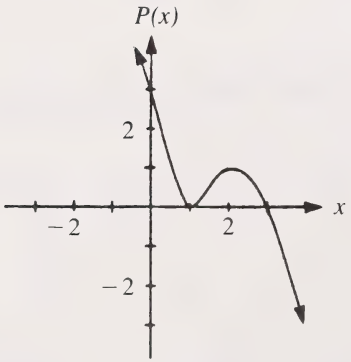
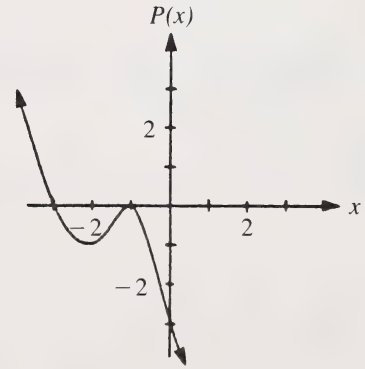


D.



YOU HAVE NOW COMPLETED THE MULTIPLE-CHOICE SECTION OF THE EXAMINATION. PLEASE PROCEED TO THE NEXT PAGE AND ANSWER THE WRITTEN-RESPONSE QUESTIONS IN PART B.

PART B

INSTRUCTIONS

Please write your answers in the examination booklet as neatly as possible.

Show all pertinent calculations and formulas.

NOTE: The perforated pages at the back of this booklet may be torn out and used for your rough work.

TOTAL MARKS: 13

START PART B IMMEDIATELY

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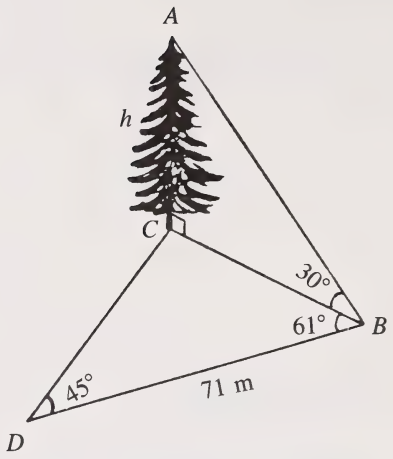
(5 marks)

1. a. Use the information in the diagram at the right to calculate the height of the tree. (Answer to one decimal place.)



b. What is the area of ΔABC? (Answer to one decimal place.)

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2. a. Find the equation of a circle that has its centre at (3, 5) and passes through the point $(-2, 9)$.

(4 marks)

b. How does the radius of this circle compare with the radius of the circle $x^2 + y^2 - 2x - 2y - 39 = 0$? (Show your work.)

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(4 marks)

3. **a.** Cucumbers chosen at random have a mean mass of 300 g and a standard deviation of 75 g. Assuming normal distribution, what is the probability of choosing a cucumber with a mass between 150 g and 375 g?

b. In a batch of 20 000 cucumbers, how many could be expected to fall within this range?

YOU HAVE NOW COMPLETED THE EXAMINATION. IF YOU HAVE TIME, YOU MAY WISH TO GO BACK AND CHECK YOUR ANSWERS.

(NO MARKS WILL BE GIVEN FOR WORK DONE ON THIS PAGE)

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M1 M2 M3 M4

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MATHEMATICS 30

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